$\boxed{M1,\ I\ колокв.\ |\ Група\ E}$

$x, y \in A \Rightarrow$

(1.) $\Rightarrow x > -3,\ y > -3$

a) затвореност

$x * y = 2x(y+3) + 6y + 18 - 3 = (2x+6)(y+3) - 3 =$

$= \underbrace{2(x+3)(y+3)}_{>0} - 3 > -3 \Rightarrow x * y \in A$ $*$ је затвор.

3

b) асоцијативност

$(x*y)*z = x*(y*z)$; $L = (2(x+3)(y+3) - 3) * z = 2(2xy + 6x + 6y + 15)\cdot z$

$+6(2xy + 6x + 6y + 15) + 6z + 15 = 4xyz + 12xz + 12yz + 30z$

$+12xy + 36x + 36y + 90 + 6z + 15 = 4xyz + 12(xy + xz + yz)$

$+36(x+y+z) + 105$

6

$D = x*(2yz + 6y + 6z + 15) = 2x(2yz + 6y + 6z + 15) + 6x +$

$+6(2yz + 6y + 6z + 15) + 15 = 4xyz + 12xy + 12xz + 30x + 6x$

$+12yz + 36y + 36z + 90 + 15 = 4xyz + 12(xy + xz + yz)$

$+36(x+y+z) + 105$ $L = D$ важи асоц.

4

c) комутативност $\quad x*y = y*x$ важи

$2xy + 6x + 6y + 15 = 2yx + 6y + 6x + 15$

3

c) неутрал

$x * e = e * x = x$

4

$2xe + 6x + 6e + 15 = x \qquad (x+3)(2e+5) = 0 \Rightarrow 2e+5 = 0$

$x(2e+5) + 6e + 15 = 0 \qquad \boxed{e = -\frac{5}{2}}$

d) инверзни ел.

$x * y = e$

4

$2xy + 6x + 6y + 15 = -\frac{5}{2}$

$2(x+3)(y+3) - 3 = -\frac{5}{2} \Rightarrow 2(x+3)(y+3) = \frac{1}{2}$

$y + 3 = \frac{1}{4(x+3)} \Rightarrow \boxed{y = \frac{1}{4(x+3)} - 3}$

(5.) $n_\alpha = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{vmatrix} = 5(\vec{i} + \vec{j} + \vec{k})$ 9

$\alpha : x + y + z + D = 0$ 12 $\qquad \boxed{\alpha : x + y + z + 3 = 0}$

$P(1, -3, -1) \in \alpha \qquad Q(2, -1, 0) \notin \alpha$ јер: 3

$\Rightarrow 1 - 3 - 1 + D = 0 \Rightarrow D = 3$ 6 $\qquad 2 - 1 + 0 + 3 = 4 \neq 0$

2.

$$\begin{bmatrix}3&1&1&4\\ \lambda&4&10&1\\ 1&7&17&3\\ 2&2&4&3\end{bmatrix}\sim\begin{bmatrix}1&1&4&3\\ 4&10&1&\lambda\\ 7&17&3&1\\ 2&4&3&2\end{bmatrix}\sim\begin{bmatrix}1&1&4&3\\ 2&4&3&2\\ 7&17&3&1\\ 4&10&1&\lambda\end{bmatrix}\begin{matrix}(-2)(-7)(-4)\end{matrix}\sim$$

$$\sim\begin{bmatrix}1&1&4&3\\ 0&2&-5&-4\\ 0&10&-25&-20\\ 0&6&-15&\lambda-12\end{bmatrix}\begin{matrix}(-5)(-3)\end{matrix}\sim\begin{bmatrix}1&1&4&3\\ 0&2&-5&-4\\ 0&0&0&0\\ 0&0&0&\lambda\end{bmatrix}$$

1° $\lambda=0$, $rA=2$

2° $\lambda\neq 0$ $rA=3$

3.

$$\left[\begin{array}{ccc|c}1&-3&-1&1\\ 3&-10&-6&2\\ 1&-2&2&2\\ -2&3&-7&\varepsilon\end{array}\right]\begin{matrix}(-3)(-1)\,2\end{matrix}\sim\left[\begin{array}{ccc|c}1&-3&-1&1\\ 0&-1&-3&-1\\ 0&1&3&1\\ 0&-3&-9&\varepsilon+2\end{array}\right]\begin{matrix}+\,(-3)\end{matrix}\sim$$

$$\left[\begin{array}{ccc|c}1&-3&-1&1\\ 0&-1&-3&-1\\ 0&0&0&0\\ 0&0&0&\varepsilon+5\end{array}\right]\ 5$$

1° $\varepsilon=5$, $rA=rA^*=2$ 1 парам. 3

2° $\varepsilon\neq 5$, $rA=2$, $rA^*=3$ нема реш. 3

$\varepsilon=5!$ $x-3y-z=1$ $x=1+3y+z$

$-y+3z=-1 \Rightarrow y=1-3z$

$(x,y,z)\in\{(1+3(1-3\alpha)+\alpha,\ 1-3\alpha,\ \alpha),\ \alpha\in\mathbb{R}\}$ 9

$(4-8\alpha,\ 1-3\alpha,\ \alpha)$

4.

a) $\overrightarrow{BA}=(2,-5,4)$, $\overrightarrow{BC}=(-2,-4,2)$, $\overrightarrow{BD}=(3,0,3)$ 2

$$6V_{\triangle}=\left|\begin{vmatrix}2&-5&4\\ -2&-4&2\\ 3&0&3\end{vmatrix}\right|=\left|-6\begin{vmatrix}2&-5&4\\ 1&2&-1\\ 1&0&1\end{vmatrix}\right|$$ 7

$$V=\left|-\left(\begin{vmatrix}-5&4\\ 2&-1\end{vmatrix}+\begin{vmatrix}2&-5\\ 1&2\end{vmatrix}\right)\right|=\left|-\left(\underbrace{5-8}_{-3}+\underbrace{4+5}_{9}\right)\right|=|-6|=6$$

b) $$P_{\triangle}=\left|\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ -2&-4&2\\ 3&0&3\end{vmatrix}\right|=\left|-6\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ 1&2&-1\\ 1&0&1\end{vmatrix}\right|=|-6(2i+2j-2k)|$$

$$=|-12|\,|\vec{i}+\vec{j}-\vec{k}|=12\cdot\sqrt{1^2+1^2+1^2}=12\sqrt{3}$$ $\cancel{1}9(\cancel{7}+2)$

$$H_A=\frac{6V}{P}=\frac{36}{12\sqrt{3}}=\frac{3}{\sqrt{3}}=\sqrt{3}$$ 2