I колокв. М1 група М

1.) a)

$$M_1 \cdot M_2 = \begin{bmatrix} u_1 & 0 & w_1 \\ u_1 & 0 & w_1 \\ -w_1 & 0 & u_1 \end{bmatrix} \cdot \begin{bmatrix} u_2 & 0 & w_2 \\ u_2 & 0 & w_2 \\ -w_2 & 0 & u_2 \end{bmatrix} = \begin{bmatrix} u_1u_2 - w_1w_2 & 0 & u_1w_2 + w_1u_2 \\ u_1u_2 - w_1w_2 & 0 & u_1w_2 + w_1u_2 \\ -w_1u_2 - u_1w_2 & 0 & -w_1w_2 + u_1u_2 \end{bmatrix}$$

$= M$ $\quad u = u_1u_2 - w_1w_2$

$w = u_1w_2 + w_1u_2$

$(u_1, w_1) \neq (0,0)$, $(u, v) = (0,0)$ 3

$$D = \begin{vmatrix} u_1 & -w_1 \\ w_1 & u_1 \end{vmatrix} = u_1^2 + w_1^2 \neq 0$$

$$D_{u_2} = \begin{vmatrix} 0 & -w_1 \\ 0 & u_1 \end{vmatrix} = 0, \quad D_{w_2} = 0$$

$\Rightarrow (u_2, w_2) = 0 \quad \perp \Rightarrow (u, v) \neq (0,0)$

• операција је затв. на скупу М 3

б) Асоц. важи у општем случ. множ. матрица

$(M_1 \cdot M_2) \cdot M_3 = M_1 \cdot (M_2 \cdot M_3)$ 3

е) комутат. $M_1 \cdot M_2 = M_2 \cdot M_1$ $= M_{u_1u_2 - w_1w_2,\, u_1w_2 + w_1u_2}$

важи јер $= M_{u_2u_1 - w_2w_1,\, u_2w_1 + w_2u_1}$

3

c) неутр. $M \cdot E = E \cdot M = M$

$$M_{u,w} \cdot E_{c,f} = M_{uc - wf,\, uf + cw} = M_{u,w}$$

$uc - wf = u$ $\quad c = 1,\ f = 0$

$uf + cw = w$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ неутр. ел.

4

f) инверз. $M_{u,w} \cdot M^{-1}_{a,b} = M^{-1} \cdot M = E_{1,0}$

$ua - wb = 1$

$ub + wa = 0$

$$D = \begin{vmatrix} u & -w \\ w & u \end{vmatrix} = u^2 + w^2$$

$$D_a = \begin{vmatrix} 1 & -w \\ 0 & u \end{vmatrix} = u, \quad D_b = \begin{vmatrix} u & 1 \\ w & 0 \end{vmatrix} = -w$$

$$M^{-1} = \frac{1}{u^2 + w^2} \begin{bmatrix} u & 0 & w \\ u & 0 & w \\ -w & 0 & u \end{bmatrix}$$ 4

5.) α:

$$\begin{vmatrix} x & y-2 & z+1 \\ 4 & -1 & 3 \\ -4 & -2 & -3 \end{vmatrix} = x(3+6) + (y-2)\underbrace{(-12+12)}_{0} + (z+1)(-8-4)$$

3

$= 9x - 12z - 12 = 0$, $3x - 4z - 4 = 0$

9

$$d(\alpha, D) = \frac{|3 \cdot 1 + 0 \cdot (-3) - 4 \cdot 1 - 4|}{\sqrt{9 + 0 + 16}} = \frac{|-5|}{5} = 1$$

1 2 1

② $X\cdot A+B=C \qquad A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$

$XA=C-B \qquad B=\begin{bmatrix}1 & 1\\ 3 & -4\\ -2 & 3\end{bmatrix} \qquad C=\begin{bmatrix}2 & 0\\ 3 & 1\\ 0 & 1\end{bmatrix}$

$X=(C-B)/\cdot A^{-1}$

$\underbrace{C-B}_{D}=\begin{bmatrix}1 & -1\\ 0 & 5\\ 2 & -2\end{bmatrix}_3, \quad A^T=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}^1 \quad A_{adj}=\begin{bmatrix}3 & 1\\ -2 & 1\end{bmatrix}_6$

$\det A=3+2=5 \quad 2$

$$\underline{\underline{X}}=D\cdot A^{-1}=\begin{bmatrix}1 & -1\\ 0 & 5\\ 2 & -2\end{bmatrix}\cdot\frac{1}{5}\cdot\begin{bmatrix}3 & 1\\ -2 & 1\end{bmatrix}=\frac{1}{5}\begin{bmatrix}5 & 0\\ -10 & 5\\ 10 & 0\end{bmatrix}=\underline{\underline{\begin{bmatrix}1 & 0\\ -2 & 1\\ 2 & 0\end{bmatrix}}}^{8}$$

③ $$\left[\begin{array}{ccc|c}1 & 1 & 2 & 0\\ -1 & 0 & 2 & -5\\ 3 & 2 & \mu & 5\end{array}\right]\sim\left[\begin{array}{ccc|c}1 & 1 & 2 & 0\\ 0 & 1 & 4 & -5\\ 0 & -1 & \mu-6 & 5\end{array}\right]\sim\left[\begin{array}{ccc|c}1 & 1 & 2 & 0\\ 0 & 1 & 4 & -5\\ 0 & 0 & \mu-2 & 0\end{array}\right]_5$$

$1^\circ\ \mu=2 \quad rA=rA^*=2$ 1 парам реш. 3

$2^\circ\ \mu\neq 2 \quad rA=rA^*=3$ јединств. реш. 3

$1^\circ\ x+y+2z=0$

$y+4z=-5 \underset{z=\alpha}{\Rightarrow} y=-5-4\alpha, \quad x=-y-2z=5+4\alpha-2\alpha$
$=5+2\alpha$

$(x,y,z)=(5+2\alpha,-5-4\alpha,\alpha)$ 5

$2^\circ\ z=0 \Rightarrow y=-5, \ x=5 \qquad (x,y,z)=(5,-5,0)$ 4

④ $\alpha a+\beta b+\gamma c=0$

$\alpha(3,m+2,5)+\beta(2,3,1)+\gamma(m+1,9,3)=(0,0,0)$ 2

$3\alpha+2\beta+(m+1)\gamma=0$
$(m+2)\alpha+3\beta+9\gamma=0$
$5\alpha+\beta+3\gamma=0$ 1

$$\begin{bmatrix}3 & 2 & m+1\\ m+2 & 3 & 9\\ 5 & 1 & 3\end{bmatrix}\sim\begin{bmatrix}5 & 1 & 3\\ m+2 & 3 & 9\\ 3 & 2 & m+1\end{bmatrix}\sim$$

$$\sim\begin{bmatrix}1 & 5 & 3\\ 3 & m+2 & 9\\ 2 & 3 & m+1\end{bmatrix}\sim\begin{bmatrix}1 & 5 & 3\\ 0 & m-13 & 0\\ 0 & -7 & m-5\end{bmatrix}\sim\begin{bmatrix}1 & 3 & 5\\ 0 & m-5 & -7\\ 0 & 0 & m-13\end{bmatrix}_5$$

$1^\circ\ m=5^4$ или $m=13^4 \quad rA=2$ зависни су

$2^\circ\ m\neq 5$ и $m\neq 13 \quad rA=3$ независни су 4