М1 , I кол. Г

1. $\mathcal{M} = \left\{ \begin{bmatrix} 0 & -x \\ 0 & x \end{bmatrix} : x \in R \right\}$

1° затвореност:

$$\begin{bmatrix} 0 & -x \\ 0 & x \end{bmatrix} \cdot \begin{bmatrix} 0 & -y \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & -xy \\ 0 & xy \end{bmatrix} \in \mathcal{M} \quad \text{јер} \quad xy \in R$$ 3

2° асоцијативност

Асоцијативност множења матрица важи за све матрице 2

3° неутрални елемент:

$$\begin{bmatrix} 0 & -x \\ 0 & x \end{bmatrix} \cdot \begin{bmatrix} 0 & -y \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & -x \\ 0 & x \end{bmatrix} \Leftrightarrow \left.\begin{matrix} -xy = -x \\ xy = x \end{matrix}\right\} \Leftrightarrow x(y-1)=0 \Rightarrow y=1$$

$$E = \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$$ 4

4° $A_x = \begin{bmatrix} 0 & -x \\ 0 & x \end{bmatrix}$

а) $x \neq 0 \Rightarrow A_x \cdot A_{\frac{1}{x}} = \begin{bmatrix} 0 & -x \\ 0 & x \end{bmatrix} \cdot \begin{bmatrix} 0 & -\frac{1}{x} \\ 0 & \frac{1}{x} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix} = E \Rightarrow$

$$A_x^{-1} = A_{\frac{1}{x}} = \begin{bmatrix} 0 & -\frac{1}{x} \\ 0 & \frac{1}{x} \end{bmatrix}$$ 3

б) $x=0$: A_0 нема инверзни елемент 3

5° комутативност:

$$A_x \cdot A_y = \begin{bmatrix} 0 & -xy \\ 0 & xy \end{bmatrix} = A_y \cdot A_x \quad \checkmark$$ 2

Структура $(\mathcal{M}, \cdot)$ није група (самим тим ни Абелова). 3

2 $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 \\ 5 & 2 \\ -2 & 1 \end{bmatrix}$

$$2A + B = \begin{bmatrix} 2 & 4 \\ 6 & -2 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix} \quad 2$$

$\det(2A+B) = \begin{vmatrix} 4 & 1 \\ 5 & 2 \end{vmatrix} = 3 \neq 0$; матрица $2A+B$ јесте регуларна 4

$$2XA = C - XB$$

$$2XA + XB = C$$

$$X(2A+B) = C \Rightarrow X = C \cdot (2A+B)^{-1} \quad 3$$

$\operatorname{cof}(2A+B) = \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix}$; $\operatorname{adj}(2A+B) = \begin{bmatrix} 2 & -1 \\ -5 & 4 \end{bmatrix}$; $(2A+B)^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -5 & 4 \end{bmatrix}$ 5

$$X = \begin{bmatrix} 1 & -2 \\ 5 & 2 \\ -2 & 1 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -5 & 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 12 & -9 \\ 0 & 3 \\ -9 & 6 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 0 & 1 \\ -3 & 2 \end{bmatrix} \quad 6$$

3

$$\begin{aligned} x + y + z + 2w &= 0 \\ -x + y + 2w &= -5 \\ 3x + y + 2z + \Gamma \cdot w &= 5 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ -1 & 1 & 0 & 2 & -5 \\ 3 & 1 & 2 & \Gamma & 5 \end{array}\right] \begin{matrix} \\ + \\ \cdot(-3) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 4 & -5 \\ 0 & -2 & -1 & \Gamma-6 & 5 \end{array}\right] + \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 4 & -5 \\ 0 & 0 & 0 & \Gamma-2 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{cccc} x & y & w & z \end{array} \\ \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 2 & 4 & 1 & -5 \\ 0 & 0 & \Gamma-2 & 0 & 0 \end{array}\right] \end{array} \quad 4$$

1° $\Gamma \neq 2$: $r(A) = r(A|B) = 3$ 3

$$\begin{aligned} x + y + 2w + z &= 0 \\ 2y + 4w + z &= \cancel{0}\,5 \\ (\Gamma-2)w &= 0 \end{aligned}$$

$z = \alpha$:

$w = 0$

$y = \cancel{-\frac{1}{2}\alpha}\ \frac{-5-\alpha}{2}$

$x = \frac{5-\alpha}{2}$

$$\{(x,y,z)\} = \left\{\left(\frac{5}{2} - \frac{\alpha}{2}, -\frac{5}{2} - \frac{\alpha}{2}, \alpha, 0\right) \middle| \alpha \in R\right\} \quad 5$$

2° $\Gamma = 2$: $r(A) = r(A|B) = 2$ 3

$$\begin{aligned} x + y + 2w + z &= 0 \\ 2y + 4w + z &= \cancel{0}\ -5 \end{aligned}$$

$w = \alpha$, $z = \beta$:

$x + y = -2\alpha - \beta$ $\Rightarrow x = \frac{5-\beta}{2}$

$2y = 5 - 4\alpha - \beta$ $\Rightarrow$ $y = -\frac{5+4\alpha+\beta}{2}$

$$\{(x,y,z)\} = \left\{\left(\frac{5-\beta}{2}, -\frac{5+4\alpha+\beta}{2}, \beta, \alpha\right) \middle| \alpha, \beta \in R\right\} \quad 5$$

4

$$a = 3e_1 + me_2 + 3e_3$$
$$b = me_1 + 5e_2 + 4e_3$$
$$c = e_1 + 3e_2 + 2e_3$$

$$\lambda_1 a + \lambda_2 b + \lambda_3 c = 0 \iff (3\lambda_1 + m\lambda_2 + \lambda_3)e_1 + (m\lambda_1 + 5\lambda_2 + 3\lambda_3)e_2 + (3\lambda_1 + 4\lambda_2 + 2\lambda_3)e_3 = 0$$

4

$$\iff \begin{aligned} 3\lambda_1 + m\lambda_2 + \lambda_3 &= 0 \\ m\lambda_1 + 5\lambda_2 + 3\lambda_3 &= 0 \\ 3\lambda_1 + 4\lambda_2 + 2\lambda_3 &= 0 \end{aligned} \quad ; \quad D = \begin{vmatrix} 3 & m & 1 \\ m & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = -2m^2 + 13m - 21 = 0$$

$$\iff m = 3 \lor m = \frac{7}{2}$$ 4

1° $m \neq 3 \land m \neq \frac{7}{2} \Rightarrow D \neq 0 \Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0$

6 вектори a, b и c су линеарно независни

2° $m = 3 \lor m = \frac{7}{2} \Rightarrow D = 0 \Rightarrow \lambda_1^2 + \lambda_2^2 + \lambda_3^2 > 0$

6 вектори a, b и c су линеарно зависни

5

$$\measuredangle(p, q) = \measuredangle(\vec{e_p}, \vec{e_q})$$

$$\vec{e_p} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix} = \vec{i} + \vec{j} \quad ; \quad \vec{e_p} = (1, 1, 0)$$ 8

$$\vec{e_q} = (1, 2, 1)$$

$$\cos(\vec{e_p}, \vec{e_q}) = \frac{\vec{e_p} \cdot \vec{e_q}}{|\vec{e_p}| \cdot |\vec{e_q}|} = \frac{\overset{1+2}{\cancel{1+3+0}}}{\sqrt{1+1} \cdot \sqrt{1+4+1}} = \frac{3}{\sqrt{2} \cdot \sqrt{6}} = \cancel{\frac{5}{\sqrt{12}}}$$ 12

$$= \frac{\sqrt{3}}{2} \Rightarrow \varphi = \frac{\pi}{6} \quad (= 30^\circ)$$