2. ГРУПА

1. $\frac{\partial}{\partial y}\left[2\lambda(y)(x+2\sin y)\right]=\frac{\partial}{\partial x}\left[-\lambda(y)(x^2+1)\operatorname{ctg}y\right] \Rightarrow \dots \Rightarrow \lambda'(y)\cancel{(x+2\sin y)}=-\frac{\cos y}{\sin y}\lambda(y)\cdot\cancel{(2\sin y+x)} \Rightarrow$

$\int\frac{d\lambda(y)}{\lambda(y)}=-\int\frac{\cos y}{\sin y}dy \Rightarrow \dots \Rightarrow \lambda(y)=\sin^{-1}y=\frac{1}{\sin y}$

$\Rightarrow 2\left(\frac{x}{\sin y}+2\right)dx-\frac{(x^2+1)\cos y}{\sin^2 y}\cdot dy=0$ једн. са тот. диф. 33б.

$\Rightarrow u(x,y)=\int P\,dx+\int\left[Q-\frac{\partial}{\partial y}\int P\,dx\right]dy=\dots=\frac{x^2+1}{\sin y}+4x \Rightarrow \frac{x^2+1}{\sin y}+4x=C$

2. $\lambda^3+\lambda=\lambda(\lambda^2+1) \Rightarrow \lambda_1=0,\ \lambda_{2,3}=\pm i \Rightarrow y_H=C_1+C_2\cos x+C_3\sin x$

$$\left.\begin{aligned} C_1'+C_2'\cos x+C_3'\sin x&=0\\ -C_2'\sin x+C_3'\cos x&=0\\ -C_2'\cos x-C_3'\sin x&=\operatorname{tg}x\end{aligned}\right\} \Rightarrow \dots \Rightarrow$$

или $\ln\left|\frac{1+\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}\frac{x}{2}}\right|$

$$\Rightarrow\left\{\begin{aligned} C_3'&=-\frac{\sin^2 x}{\cos x} \Rightarrow C_3=C_3(x)=\sin x-\frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right|+D_3\\ C_2'&=-\sin x \Rightarrow C_2=C_2(x)=\cos x+D_2\\ C_1'&=\frac{\sin x}{\cos x} \Rightarrow C_1=C_1(x)=-\ln|\cos x|+D_1\end{aligned}\right\} \Rightarrow \begin{aligned} &y=C_1(x)+C_2(x)\cos x+C_3(x)\sin x\\ &y=\dots\end{aligned}$$

33б.

3. $A=\begin{bmatrix}-1&2&1\\0&2&1\\-2&-2&-1\end{bmatrix} \Rightarrow \det(A-\lambda I)=\begin{vmatrix}-1-\lambda&2&1\\0&2-\lambda&1\\-2&-2&-1-\lambda\end{vmatrix}=(\lambda+1)^2(2-\lambda)-4+2(2-\lambda)-2(\lambda+1)=\dots=-\lambda(\lambda^2+1)$

$\Rightarrow \det(A-\lambda I)=0 \Leftrightarrow \lambda_1=0,\ \lambda_{2,3}=\pm i$

34б.

4. ГРУПА

$\boxed{1}$ $\frac{\partial}{\partial y}\left[\lambda(x)(y^2+1)\,\mathrm{ctg}\,x\right] = \frac{\partial}{\partial x}\left[-2(y+2\sin x)\cdot\lambda(x)\right] \Rightarrow \ldots \Rightarrow \lambda'(x)\cancel{(y+2\sin x)} = -\lambda(x)\cdot\frac{\cos x}{\sin x}\cdot\cancel{(y+2\sin x)} \Rightarrow$

$\int\frac{d\lambda(x)}{\lambda(x)} = -\int\frac{\cos x}{\sin x}dx \Rightarrow \ldots \Rightarrow \underline{\lambda(x) = \sin^{-1}x = \frac{1}{\sin x}}$

$\Rightarrow \frac{(y^2+1)\cos x}{\sin^2 x}dx - 2\left(\frac{y}{\sin x}+2\right)dy = 0$ једн. са тот. диф.

$\Rightarrow u(x,y) = \int P dx + \int\left[Q - \frac{\partial}{\partial y}\int P dx\right]dy = \ldots = -\frac{y^2+1}{\sin x} - 4y \Rightarrow \underline{\frac{y^2+1}{\sin x} + 4y = C}$

$\boxed{2}$ $\lambda^3+\lambda = \lambda(\lambda^2+1) \Rightarrow \lambda_1 = 0,\ \lambda_{2,3} = \pm i \Rightarrow y_H = C_1 + C_2\cdot\cos x + C_3\cdot\sin x$

$$\left.\begin{aligned} C_1' + C_2'\cdot\cos x + C_3'\cdot\sin x &= 0 \\ -C_2'\cdot\sin x + C_3'\cdot\cos x &= 0 \\ -C_2'\cdot\cos x - C_3'\cdot\sin x &= \mathrm{ctg}\,x \end{aligned}\right\} \Rightarrow \ldots \Rightarrow$$

или $-\ln\left|\mathrm{tg}\frac{x}{2}\right|$

$$\Rightarrow \left\{\begin{aligned} C_3' &= -\cos x &&\Rightarrow \underline{C_3} = C_3(x) = -\sin x + D_3 \\ C_2' &= -\frac{\cos^2 x}{\sin x} &&\Rightarrow \underline{C_2} = -\cos x + \frac{1}{2}\ln\left|\frac{1+\cos x}{1-\cos x}\right| + D_2 \\ C_1' &= \frac{\cos x}{\sin x} &&\Rightarrow \underline{C_1} = \ln|\sin x| + D_1 \end{aligned}\right\} \Rightarrow \begin{aligned} y &= C_1(x) + C_2(x)\cdot\cos x + C_3(x)\cdot\sin x \\ y &= \ldots \end{aligned}$$

$\boxed{3}$ $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 0 \\ 1 & 2 & -1 \end{bmatrix} \Rightarrow \det(A-\lambda I) = \begin{vmatrix} -1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 0 \\ 1 & 2 & -1-\lambda \end{vmatrix} = (\lambda+1)^2(2-\lambda) - 4 + 2(2-\lambda) - 2(\lambda+1) = \ldots = -\lambda(\lambda^2+1)$

$\Rightarrow \det(A-\lambda I) = 0 \Leftrightarrow \underline{\lambda_1 = 0,\ \lambda_{2,3} = \pm i}$