1. GRUPA

1.) $\left(\frac{x^2y}{\sqrt{1+x^2}}+2x^2y-y\right)dx+\left(x\sqrt{1+x^2}+x^3-x\ln x\right)dy=0$

$\lambda=\lambda(x)$

$P=\frac{x^2y}{\sqrt{1+x^2}}+2x^2y-y$

$(\lambda\cdot P)'_y=(\lambda Q)'_x$

$Q=x\sqrt{1+x^2}+x^3-x\ln x$

$\lambda(x)\cdot P'_y=\lambda'(x)\cdot Q+\lambda\cdot Q'_x$

$\lambda(x)\cdot\left(\frac{x^2}{\sqrt{1+x^2}}+2x^2-1\right)=\lambda'_x\cdot\left(x\sqrt{1+x^2}+x^3-x\ln x\right)+\lambda\left(\sqrt{1+x^2}+x\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}+3x^2-\ln x-x\cdot\frac{1}{x}\right)$

$\lambda(x)\cdot\left(\cancel{\frac{x^2}{\sqrt{1+x^2}}}+2x^2-\cancel{1}-\sqrt{1+x^2}-\cancel{\frac{x^2}{\sqrt{1+x^2}}}-3x^2+\ln x+\cancel{1}\right)$

$=\lambda'_x\left(x\sqrt{1+x^2}+x^3-x\ln x\right)$

$\frac{\lambda'}{\lambda}=\frac{\ln x-x^2-\sqrt{1+x^2}}{x\sqrt{1+x^2}+x^3-x\ln x}$

$\frac{\lambda'}{\lambda}=\frac{\ln x-x^2-\sqrt{1+x^2}}{-x\left(-\sqrt{1+x^2}-x^2+\ln x\right)}$

$\frac{\lambda'}{\lambda}=-\frac{1}{x}\longrightarrow\frac{d\lambda}{\lambda}=-\frac{dx}{x}\Big/\int$

$\int\frac{d\lambda}{\lambda}=\int-\frac{dx}{x}$

$\ln|\lambda|=-\ln|x|+c$

$\ln|\lambda x|=c$

$\lambda x=c$

$\lambda=\frac{c}{x}$ npr. $c=1$

$\boxed{\lambda=\frac{1}{x}}$

$$\lambda P\, dx + \lambda Q\, dy = 0$$

$$(\lambda P)'_y = (\lambda Q)'_x \longrightarrow$$ JEDNAČINA SA TOTALNIM DIFERENCIJALOM

$$\Big(\underbrace{\frac{xy}{\sqrt{1+x^2}} + 2xy - \frac{y}{x}}_{\lambda P}\Big)dx + \Big(\underbrace{\sqrt{1+x^2} + x^2 - \ln x}_{\lambda Q}\Big)dy = 0$$

$$\boxed{\int(\lambda P)dx + \int\Big(\lambda Q - \frac{\partial}{\partial y}\int(\lambda P)\,dx\Big)dy = C}$$

$$\int(\lambda P)dx = \int\Big(\frac{xy}{\sqrt{1+x^2}} + 2xy - \frac{y}{x}\Big)dx$$

$$= y\int\frac{\frac{1}{2}\cdot 2x\,dx}{\sqrt{1+x^2}} + 2\cdot\frac{x^2}{2}\cdot y - y\ln x$$

$$= \frac{1}{2}y\cdot\int\frac{d(1+x^2)}{\sqrt{1+x^2}} + x^2y - y\ln x$$

$$= \cancel{\frac{1}{2}}y\cdot\frac{\sqrt{1+x^2}}{\cancel{\frac{1}{2}}} + x^2y - y\ln x$$

$$\frac{\partial}{\partial y}\int(\lambda P)dx = \sqrt{1+x^2} + x^2 - \ln x$$

$$\lambda Q - \frac{\partial}{\partial y}\int\lambda P\,dx = \cancel{\sqrt{1+x^2}} + \cancel{x^2} - \cancel{\ln x} - \cancel{\sqrt{1+x^2}} - \cancel{x^2} + \cancel{\ln x}$$

$$= 0$$

$$\int 0\,dy = 0 + C_1$$

$$\int(\lambda P)\,dx + \int\Big(\lambda Q - \frac{\partial}{\partial y}\int(\lambda P)dx\Big)dy$$

$$= y\sqrt{1+x^2} + x^2y - y\ln x + C_1 = C$$

$$\boxed{y\sqrt{1+x^2} + x^2y - y\ln x = C}$$

2.) $$\frac{dx}{y+t} = \frac{dy}{x+t} = \frac{dt}{x+y}$$

$$\frac{dx - dy}{y+\cancel{t}-x-\cancel{t}} = \frac{dy-dt}{\cancel{x}+t-\cancel{x}-y}$$

$$\frac{d(x-y)}{-(x-y)} = \frac{d(y-t)}{-(y-t)}$$

$$\frac{d(x-y)}{x-y} = \frac{d(y-t)}{y-t} \quad / \int$$

$$\ln|x-y| = \ln|y-t| + c$$

$$\ln\left|\frac{x-y}{y-t}\right| = c$$

$$\frac{x-y}{y-t} = c \longrightarrow \qquad \frac{x-y}{t-y} = -c \qquad \frac{t-y}{x-y} = -\frac{1}{c}$$

$$\mathrm{I} \quad \boxed{\frac{t-y}{x-y} = c_1}$$

$$\frac{dx+dy+dt}{y+t+x+t+x+y} = \frac{dx-dy}{-(x-y)}$$

$$\frac{d(x+y+t)}{2(x+y+t)} = -\frac{d(x-y)}{(x-y)} \quad / \int$$

$$\frac{1}{2}\ln|x+y+t| = -\ln|x-y| + c \quad / \cdot 2$$

$$\ln|x+y+t| + 2\ln|x-y| = 2c$$

$$\ln|(x+y+t)(x-y)^2| = 2c$$

$$\mathrm{II} \quad \boxed{(x+y+t)(x-y)^2 = C_2}$$

$$\varphi_1 = \frac{t-y}{x-y} = c_1 \qquad \varphi_2 = (x-y)^2(x+y+t) = c_2$$

$$\frac{\partial \varphi_1}{\partial x} = -\frac{(t-y)}{(x-y)^2} = \frac{y-t}{(x-y)^2}$$

$$\frac{\partial \varphi_1}{\partial t} = \frac{+1}{x-y}$$

$$\frac{\partial \varphi_2}{\partial x} = 2(x-y)(x+y+t) + (x-y)^2 \cdot 1$$

$$= (x-y)(2x+2y+2t+x-y)$$

$$= (x-y)(3x+y+2t)$$

$$\frac{\partial \varphi_2}{\partial t} = (x-y)^2$$

NEZAVISNOST PRVIH INTEGRALA ?

$$\begin{vmatrix} \frac{\partial \varphi_1}{\partial x} & \frac{\partial \varphi_1}{\partial t} \\ \frac{\partial \varphi_2}{\partial x} & \frac{\partial \varphi_2}{\partial t} \end{vmatrix} = \begin{vmatrix} \frac{y-t}{(x-y)^2} & \frac{1}{x-y} \\ (x-y)(3x+y+2t) & (x-y)^2 \end{vmatrix}$$

$$= \frac{y-t}{\cancel{(x-y)^2}} \cdot \cancel{(x-y)^2} - \frac{1}{\cancel{x-y}} \cdot \cancel{(x-y)}(3x+y+2t)$$

$$= \cancel{y} - t - 3x - \cancel{y} - 2t$$

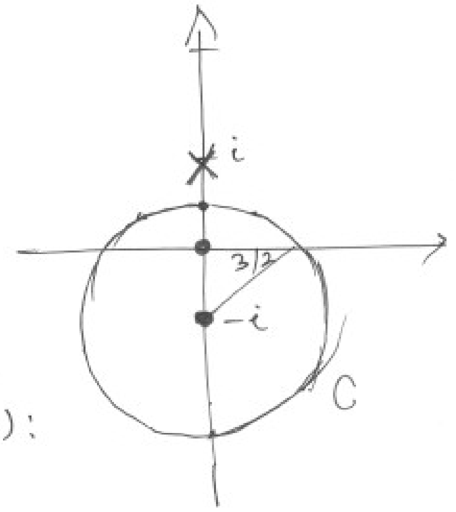
$$= -3(x+t) \neq 0$$

$$\Downarrow$$

PRVI INTEGRALI

NEZAVISNI !

3.) $\displaystyle\int_{C^+} \frac{e^z}{z^4+z^2}\,dz = ?$ $\quad C = \{ z \mid |z+i| = \frac{3}{2} \}$



$$f(z) = \frac{e^z}{z^4+z^2}$$

$$= \frac{e^z}{z^2(z^2+1)} = \frac{e^z}{z^2(z+i)(z-i)}$$

singulariteti f-je $f(z)$:

0 2. reda

i 1. reda $\longrightarrow$ nije unutar konture C

$-i$ 1. reda

$$\int_{C^+} f(z)\,dz = 2\pi i \left(\operatorname{Res}[f(z), z=0] + \operatorname{Res}[f(z), z=-i] \right)$$

$$\operatorname{Res}[f(z), z=0] = \lim_{z\to 0} \left(z^2 \cdot f(z) \right)'$$

$$= \lim_{z\to 0} \left(\cancel{z^2} \cdot \frac{e^z}{\cancel{z^2}(z^2+1)} \right)'$$

$$= \lim_{z\to 0} \left(\frac{e^z \cdot (z^2+1) - e^z \cdot 2z}{(z^2+1)^2} \right)$$

$$= \lim_{z\to 0} \frac{e^z(z^2-2z+1)}{(z^2+1)^2} = \lim_{z\to 0} \frac{e^z(z-1)^2}{(z^2+1)^2}$$

$$= \frac{e^0 \cdot (-1)^2}{1^2} = 1$$

$$\operatorname{Res}[f(z), z=-i] = \lim_{z\to -i} \left((z+i) \cdot f(z) \right)$$

$$= \lim_{z\to -i} \left(\cancel{(z+i)} \cdot \frac{e^z}{z^2 \cdot \cancel{(z+i)}(z-i)} \right)$$

$$= \frac{e^{-i}}{(-i)^2 \cdot (-i-i)} = \frac{e^{-i}}{1 \cdot (-2i)} \cdot \frac{-i}{-i}$$

$$= \frac{-i \cdot e^{-i}}{2} = \frac{1}{2}\left[-i \cdot (\cos(-1) + i\sin(-1)) \right] = \frac{1}{2}\left(\sin 1 - i\cos 1 \right)$$

$$\int_{C^+} \frac{e^z}{z^4+z^2}\,dz = 2\pi i\left(1 - \frac{i\cdot e^{-i}}{2}\right)$$
$$= 2\pi i + \pi e^{-i}$$
$$= 2\pi i + \pi(\cos 1 - i\sin 1)$$
$$= \underline{(2\pi - \sin 1)\, i + \pi\cdot\cos 1}$$

4.)
$$x' - x + y'' - y = e^{-t}$$
$$x' + x + y'' + 3y' + 2y = 2e^{t}$$

$x(0) = 0$
$y(0) = 0 \quad y'(0) = -1$

$$sX - X + s^2Y + 1 - Y = \frac{1}{s+1}$$
$$sX + X + s^2Y + 1 + 3sY + 2Y = \frac{2}{s-1}$$

$L[x] = X(s)$
$L[x'] = sX - x(0)$
$= sX - 0$
$L[x'] = sX$

$$(s-1)X + (s^2-1)Y = \frac{1}{s+1} - 1$$
$$(s+1)X + (s^2+3s+2)Y = \frac{2}{s-1} - 1$$

$L[y] = Y$
$L[y'] = sY$
$L[y''] = s^2Y - sy(0) - y'(0)$
$= s^2Y + 1$

$$(s-1)X + (s-1)(s+1)Y = \frac{-s}{s+1}$$
$$(s+1)X + (s+1)(s+2)Y = \frac{3-s}{s-1}$$

$L[e^{-1\cdot t}] = \frac{1}{s+1}$
$L[e^{1\cdot t}] = \frac{1}{s-1}$

$$\Delta = \begin{vmatrix} s-1 & (s-1)(s+1) \\ s+1 & (s+1)(s+2) \end{vmatrix}$$
$$= (s-1)(s+1)(s+2) - (s+1)(s-1)(s+1)$$
$$= (s-1)(s+1)(\cancel{s}+2 - \cancel{s} - 1)$$
$$\Delta = s^2 - 1$$

$$\Delta_X = \begin{vmatrix} -\frac{s}{s+1} & (s-1)(s+1) \\ \frac{3-s}{s-1} & (s+1)(s+2) \end{vmatrix} = -s(s+2) - (3-s)(s+1)$$
$$= -\cancel{s^2} - 2s - 3s - 3 + \cancel{s^2} + s$$
$$= -4s - 3$$

$$\Delta_y = \begin{vmatrix} s+1 & \dfrac{3-s}{s-1} \end{vmatrix}$$

$$X = \frac{\Delta x}{\Delta} = \frac{-4s-3}{s^2-1}$$

$$Y = \frac{\Delta y}{\Delta} = \frac{3}{s^2-1}$$

$$X = -4 \cdot \frac{s}{s^2-1} - 3 \cdot \frac{1}{s^2-1}$$

$$L^{-1}[X] = L^{-1}\left[-4 \cdot \frac{s}{s^2-1}\right] + L^{-1}\left[-3 \cdot \frac{1}{s^2-1}\right]$$

$$x(t) = -4 \cdot L^{-1}\left[\frac{s}{s^2-1}\right] - 3 \cdot L^{-1}\left[\frac{1}{s^2-1}\right]$$

$$\boxed{x(t) = -4 \cdot \operatorname{ch} t - 3 \cdot \operatorname{sh} t}$$

$$L^{-1}[Y] = L^{-1}\left[\frac{3}{s^2-1}\right]$$

$$y(t) = 3 \cdot L^{-1}\left[\frac{1}{s^2-1}\right]$$

$$\boxed{y(t) = 3 \cdot \operatorname{sh} t}$$